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SUMMARY

A theoretical and experimental study is in progress of the flow through slotted walls under a variety of conditions. The ultimate objective is to make possible accurate numerical computation of transonic flows around models in slotted test sections. This paper is concerned with a slot flow configuration typical of two-dimensional, low lift tests at high subsonic free stream Mach numbers. With the test section empty the slot flow is outwards, into the plenum chamber, and this remains true over a large part of the test section when the model is introduced. From oil flow pictures and pressure measurements in and around the slots it is concluded for the configurations investigated that the slot flow is slightly influenced by the presence of the wall boundary layer, that the flow within the slot is attached and approximately inviscid although influenced by boundary layer formation, that the flow enters the plenum chamber as a thin free jet and that the transverse velocity in the jet and slot is too large for a linear pressure drop equation to be sufficiently accurate. When the slot flow turns back over the rear end of the model it may admit stagnant air from the plenum chamber into the test section; the ability of the slot to maintain a pressure difference across the wall is then necessarily reduced. Based on these observations a tentative flow model is proposed, yielding a relationship between the pressure difference across the wall and the transverse velocity through the slots. It agrees well with the measurements as far as they go. The corresponding homogeneous wall boundary condition is of the classical inviscid type with an added quadratic cross-velocity term in the manner of W.W. Wood. Improved expressions for the coefficient of the streamline curvature term are obtained, accounting for the depth of the slot and the presence of a jet. The new boundary condition has been demonstrated to be compatible with computing numerically the transonic flow field around a model in the test section.

1. THE BACKGROUND

Recent development in numerical methods for transonic flow fields has made it feasible to compute the wall interference in transonic test sections with high accuracy. This new potential for designing better test sections and for improving interference correction procedures has not been used much so far, due to the fact that we do not know the structure and properties of the wall flow well enough to define accurate wall boundary conditions for the test section flow field. It is fair to say that not much progress has been made in this respect since about 1960, when the first development period of transonic test sections came to an end (Refs. 1, 2 and 3).

Two techniques for alleviating the wall interference in transonic wind tunnels have commonly been in use since then: (i) ventilation through 'slanted' perforations and (ii) ventilation through longitudinal slots. In distinctly supersonic flow only the perforations are efficient, while at sonic and high subsonic free stream Mach numbers the slots seem superior, so the two techniques may largely be considered complementary. At very low supersonic Mach numbers, however, none of them does a satisfactory job. In addition, there are many indications that in extreme conditions, such as prevail with large models at high angles of attack, ventilated walls produce interference effects which cannot be satisfactorily eliminated by wall corrections based on conventional ideas about the wall flow. In view of the need for high Reynolds number testing at transonic speeds, a rather costly need to satisfy, there are great potential savings in developing transonic test sections to admit larger models than at present.

Much effort (Refs. 4, 5 and 6, as well as papers at this symposium) is now going into devising test section walls which can be adjusted for zero wall interference as defined on the basis of numerical flow field computations. Such walls must permit accurate control of the streamline slope or the pressure (or a combination of them) in the neighbourhood of the wall, in accordance with requirements defined in a complicated way by the free stream Mach number and by the size, shape and attitude of the model - these requirements including preferably the ability to absorb shock waves. This calls for walls of continuously controllable flow properties, such as flexible solid walls, ventilated walls of variable slot width or hole size, or porous walls with controllable, distributed suction (see also Ref. 7). The results so far are clearly encouraging, although much work no doubt remains to be done before practical solutions are reached.

In the meantime, in order to make progress towards improved transonic test sections, it is necessary to acquire a better understanding of the flow at test section walls of different types and under different working conditions. As for slotted walls, which are the subject of the present investigation, our understanding of the flow is far from complete, as has become increasingly obvious in recent years.
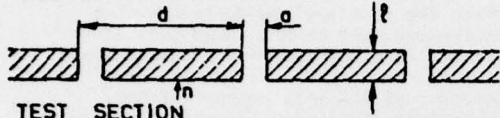
2. THE PROBLEMS

The classical method of analysing the flow in a slotted test section is to postulate small, inviscid perturbations of a homogeneous free stream and to require as boundary conditions at the wall (i) that the pressure perturbation vanishes at the boundary between the slot and the plenum chamber and (ii) that the normal velocity vanishes at solid parts. Assuming a large number of slots and applying the slender body approximation locally at the wall, these mixed boundary conditions are then usually changed, by cross-flow averaging as it were, into an equivalent homogeneous boundary condition, which produces at the centre of the test section approximately the same solution as the original set (Refs. 8-11). With the x-axis in the flow direction, the simplified boundary condition

for the perturbation velocity potential φ is of the form

$$\frac{\partial \varphi}{\partial x} + K \frac{\partial^2 \varphi}{\partial n \partial x} = 0 \quad , \tag{1}$$

where K is a positive constant determined by the geometry of the slots, while $\partial/\partial n$ denotes differentiation along the outward normal of the test section boundary. Taking the influence of the depth of the slots into account, the value of K for the geometry of Fig. 1 can be obtained approximately (Refs. 8, 10 and 12) from Eq. (2):

PLENUM

$$K = d \left(\frac{1}{\pi} \ln \frac{1}{\sin \frac{\pi a}{2d}} + \frac{\ell}{a} \right) \quad . \tag{2}$$

TEST SECTION

Fig. 1. Slot geometry
(section normal to flow direction).

The homogeneous boundary condition is essentially a balance equation in the mean between the pressure difference across the wall (proportional to $-\partial\varphi/\partial x$ in a linear approximation) and the centrifugal force due to streamline curvature in and near the slot (proportional to $\partial^2\varphi/\partial n\partial x$), so K is in effect a measure of the longitudinal momentum flux within the region of the slot flow.

Although it is believed (Refs. 1 and 2) that the wall interference predicted by (1), using values from (2) for K, is of the correct magnitude when the slots are not too narrow, the experimental evidence, largely from subsonic flows, is not really conclusive. This is because it has been common practice to design transonic test sections and models empirically so as to eliminate blockage interference at sonic speed (where, until recently, the interference could not be computed), leading to such small models as to make the subsonic interference (which could be computed) too small to be measured accurately. As already mentioned, however, at extreme conditions of model size and lift, deviations are believed to occur.

It should be stressed that any large errors inherent in (1) are likely not to arise from the averaging (as long as the number of slots is reasonably large), since, in fact, the slender body approximation, when applied locally as in the present case, is known to be particularly accurate at transonic speeds (although not sufficiently accurate, very likely, to account for shock waves reaching the wall). For similar reasons the accuracy of (1), in spite of its linear form, is fully consistent with its use as a boundary condition with the non-linear partial differential equation of transonic small perturbation theory (although this was perhaps not altogether clear in the original derivations). This leaves us to view with caution the remaining assumptions of inviscid flow, small perturbations and constant pressure (implying quiescent air) in the plenum chamber.

Quite early it was suggested (Ref. 9) that the homogeneous boundary condition should be augmented by a third term - analogous to the term expressing the linearized viscous pressure drop across a porous wall - so as to account for viscous effects in the slots:

$$\frac{\partial \varphi}{\partial x} + K \frac{\partial^2 \varphi}{\partial n \partial x} + \frac{1}{P} \frac{\partial \varphi}{\partial n} = 0 \quad . \tag{3}$$

The constant 'porosity' P was assumed to be determined by the wall geometry and the free-stream Mach number (while, in contrast to K, there is no method available for computing it a priori). It is far from obvious, however, that if viscous effects in the slots were important they would give rise to a linear cross-flow term in the boundary condition: one would rather expect a quadratic one, of course. In Ref. 2 experimental evidence is quoted which shows a clearly linear dependence (in addition to a weaker quadratic one); the set-up is such, however, as to make it improbable that viscous effects are important at all under the circumstances tested. This matter should obviously be looked into. It is also noteworthy that although many attempts have evidently been made to correlate (3) with deviations from (1), none has been reported successful. It seems, therefore, that the ideas motivating the linear porosity term in the boundary condition are probably not generally valid (although there might of course still be a pressure drop across the wall, due to viscous cross flow, which is not accounted for in (1)).

It is essential for the validity of formula (2) that the flow leaving the slot is attached; if it separates at the slot edges to form a free jet inside the slot (Fig. 2), then the term ℓ/a should be absent, and the slot would be expected to be much less efficient in maintaining a pressure difference between the test section and the plenum chamber. It seems that many people have taken it for granted that the flow is separated and, in consequence, have left out the term ℓ/a even when it would dominate the first one, were the flow attached. The fact that in such circumstances the slots appear to be much more effective than expected from the computed value for K with $\ell = 0$ (Refs. 13, 14 and 15), is a strong indication that the slot flow is in fact attached. The matter should of course be settled by direct observation.

If the slot flow, whether separated or not, penetrates the plenum chamber as a jet, as indicated in Fig. 2, then this is bound to set up a secondary flow in the plenum chamber. Obviously what is important, as far as the flow in the test section is concerned, is whether this secondary flow can change the pressure around the emerging jet.

The assumption of small perturbations (underlying the linear boundary conditions) might easily be violated inside the slots when they are narrow.
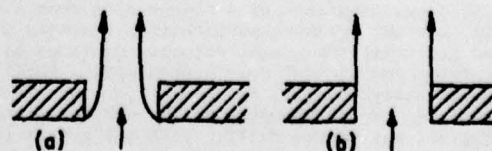

Fig. 2. Does the flow into the plenum chamber form a free jet inside the slot as in (a), or not, as in (b)?

since the cross-flow velocity in them is then bound to be very much larger than in the test section. This fact was realized at an early stage (Ref. 9) and its effect on the wall pressure was subsequently analyzed by W.W. Wood (Ref. 16). He found that it can easily be accounted for by including in the Bernoulli equation the quadratic cross-velocity term when satisfying the condition of continuous pressure at the boundary between the slot flow and the plenum air (in doing so he assumed the flow to be separated). This results in a further term in the boundary condition (1), a term proportional to the square of the cross-flow velocity. Generally speaking, there is nothing new or difficult in handling, within the slender body approximation, situations where locally the cross-flow velocity is so large as to make its square comparable to the longitudinal perturbation velocity. As we shall see, the inclusion of a quadratic cross-flow term in (1) comes quite naturally and is not much of a complication in the transonic case, where we have to solve a nonlinear problem in any event. Whether the quadratic term is important or not depends of course on the experimental circumstances. As it turns out the term is likely to be important in many practical cases, although it has in the past been rather unusual to include it when computing wall corrections.

None of the approaches to slot flow so far mentioned account for the presence of wall boundary layers. The total cross-sectional area of the boundary layers in the test section often is as large as the cross-sectional area of the model or even larger. It is possible, therefore, that the fluid going into the slots is to a considerable extent low momentum air from the boundary layer rather than free stream air. If this is so, then the slot flow is bound to be less efficient than otherwise in maintaining a pressure difference between the test section and the plenum chamber. It is one of the main purposes of the present investigation to determine the influence of the wall boundary layers on the slot flow.

These considerations apply a fortiori to a region where the slot flow passes into the test section from the plenum chamber with its more or less stagnant air. In the extreme case of vanishing longitudinal flux in the slot region there will be negligible pressure difference and the wall will appear to the test section flow as a free-jet boundary at the plenum pressure. Inflow of stagnant air is likely to occur as soon there is a model in the test section unless the walls have been adjusted to generate a sufficiently high outflow through the slots from the empty test section. The risk of extended regions of inflow of stagnant air is of course particularly great with large models at high lift, and this is perhaps the explanation of why conventional wall corrections have been found unsatisfactory in this kind of situation. It is remarkable that the idea of treating a region of inflow as a free jet boundary, although suggested early on (Refs. 1 and 8), does not seem to have been exploited in any scheme for computing the wall interference in transonic test sections.

In addition to influencing the slot flow, the wall boundary layer between slots may contribute directly to the wall interference by changing its displacement thickness in response to the pressure variation set up by the model (Ref. 17). Except where a shock wave reaches the wall, this effect is likely to be small. Nevertheless, it might be of some importance when the cross-flow component of the perturbation velocity far from the model is much smaller than the longitudinal component (i.e. when the slots are required to be narrow), as is the case with slender, small-aspect-ratio models.

Altogether, the flow properties of slotted walls might well be rather complicated, very likely more complicated than consistent with the simplifying assumptions usually made in computing the wall interference. Part of the complication is the great number of parameters involved, part derives from our not knowing what basic phenomena are important under what circumstances. However, to a large extent the basic ideas required for describing those phenomena are available and have in fact been available for a long time. What is still lacking, essentially, in order to be able to integrate these ideas into a coherent description, is experimental evidence on a small number of key issues. It is the objective of the present research to acquire such evidence by investigating in some detail a few typical cases of slot flow, and to establish, based on that evidence, a flow model capable of yielding an improved wall boundary condition for the test section flow. A first case is considered here, leading to the tentative definition of a flow model.

3. EXPERIMENTAL ARRANGEMENTS

3.1 Slot flow parameters

In order to know what constitutes a 'typical' slot flow we must define non-dimensional parameters which describe relevant local features of slot flows and then relate those to global parameters of typical test sections and models. In this way we might be able to devise a small number of experiments to simulate, locally, slot flows typical of a variety of applications.

The most important feature to be considered is whether the flow from the test section fills out the slot or not, and this is of course intimately connected with the geometrical shape of the slot. Restricting ourselves to the geometry of Fig. 1, we shall assume that by rounding the edges of the entrance we can inhibit any tendency to edge separation, thus making the radius of curvature of the edges an additional parameter to be controlled. This kind of geometry, with the slot shape the same everywhere along the test section, is commonly used in practice.

From the discussion of Section 2 it would seem that among additional features of slot flow to be considered are (i) viscous effects in the slots, (ii) the quadratic cross-flow pressure drop, (iii) the wall boundary layer going into the slots, (iv) the penetration of low momentum air from the plenum chamber into the test section and, perhaps, (v) the flow structure in the plenum chamber itself. Parameters describing these features have been defined and analysed as a basis for selecting the configurations to be investigated. However, since the results used in the present paper pertain to one particular test configuration, and since space is short, we shall not reproduce this analysis here.

The configuration with which we shall be concerned is one typical of two-dimensional tests. The slot width is somewhat smaller than the thickness of the wall boundary layer while large enough to avoid strong viscous effects in the slots. The depth of the slots is large enough for the second term in Eq. (2) to be somewhat larger than the first term. The radius of curvature of the slot edges is as small as was considered practicable. The thickness of the model (two-dimensional wing, zero lift) is somewhat smaller than the thickness of the wall boundary layers. The resulting cross-flow velocity in the slots is expected to be large enough for its square to be important. The free-stream Mach number, $M_\infty = 0.903$, is as large as possible without shock waves reaching the slotted walls. The

walls of the test section are parallel, so that with the test section empty there is a flow out through the slots in consequence of boundary layer build-up along the walls.

3.2 Equipment

The tests were made in the 23 x 25 cm^2 transonic blow-down wind tunnel (S3) of the FFA. The sketch in Fig. 3 shows the test section used in the present tests, including the model and slot arrangement. The central slot of the bottom wall was selected to be the 'test slot' and was accordingly equipped with a number of pressure taps.
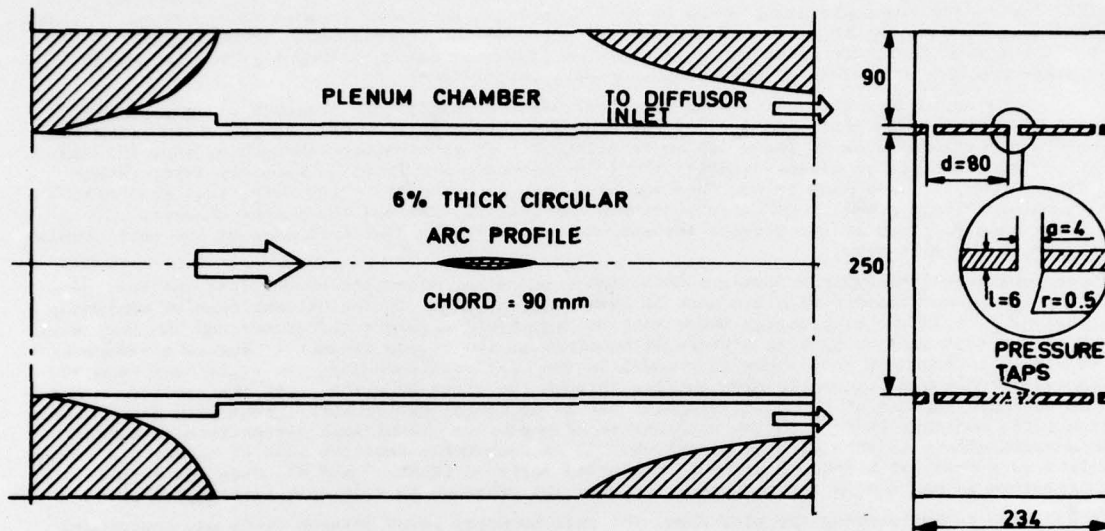

Fig. 3. Experimental configuration. (lengths in mm)

In addition to conventional pressure probes a special one has been designed for investigating the flow from the test slot into the plenum chamber. It consists of a thin circular cylinder placed close to the wall and perpendicular to the plane of symmetry of the slot (see Fig. 4). It has two pressure taps in a plane perpendicular to the axis and with an angle of 90° between them. By turning the probe until the two pressures are equal the flow direction is obtained; then turning the probe 45° gives the stagnation pressure in one of the taps. The probe can also be moved across the slot and along the slot, and all these manoeuvres can be controlled from outside the tunnel while it is running. The probe has been calibrated against conventional probes and found to be sufficiently accurate.
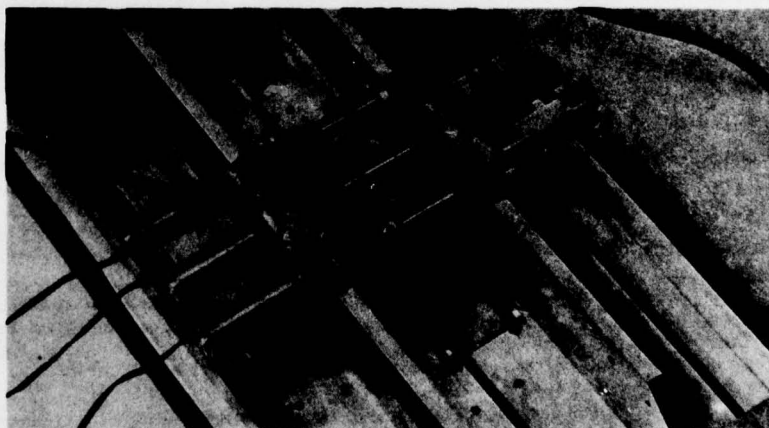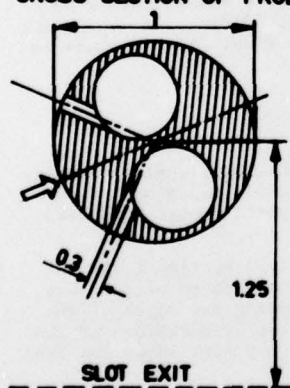

Fig. 4. Movable combined total-pressure and flow-direction probe. (lengths in mm)

4. THE STRUCTURE OF THE SLOT FLOW

4.1 Observations

Over most of the test section - with the model present as well as absent - the flow through the slots is outwards, from the test section into the plenum chamber. One expects the flow to enter the plenum chamber obliquely as a thin free jet, the thickness of which is not much different from the slot width. This is born out by the oil flow pictures of Fig. 5. The picture (a) was obtained on a thin sheet in the plane of symmetry of a slot, while the picture (b) was obtained on a sheet perpendicular to the plane of symmetry and parallel to the jet flow as indicated in (a). The deep penetration of the jet into the plenum chamber is clearly visible, as is a strong secondary flow set up in the plenum chamber by the jet, through entrainment as well as through deflection at the floor of the plenum chamber. Over the rear of the model the slot flow turns back while the free jet continues on

(a) In plane of symmetry of slot

(b) In oblique plane indicated in (a)

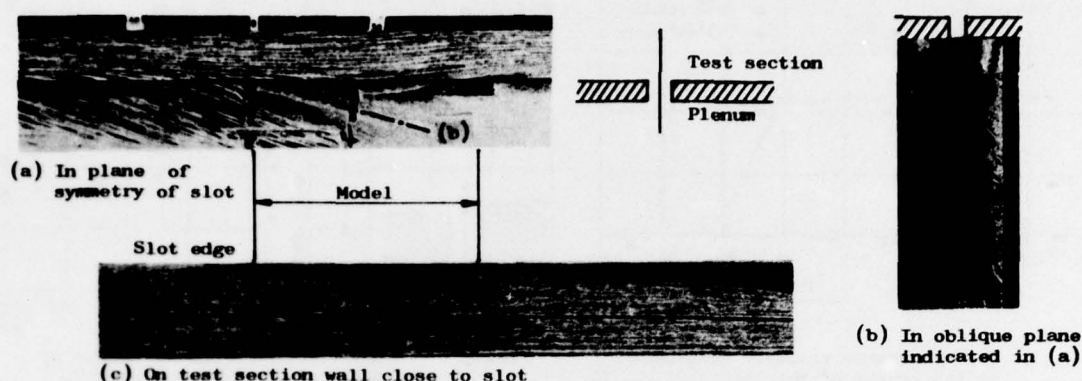(c) On test section wall close to slot

Fig. 5. Oil flow pictures of slot flow.

its own into the interior, giving stagnant air from the plenum chamber the opportunity to enter the slot and hence the test section. There is consequently an intermediate region of mixed flow in the slot consisting of both high momentum air, removed earlier from the test section, and low momentum air from the plenum chamber. The re-entering air, at least the high momentum part of it, seems perhaps to form a jet of a similar kind to that in the plenum chamber.

Further illumination is furnished by the oil flow picture (c), showing the flow over the test section wall adjacent to the slot. Where the flow goes into the slot not much is to be seen. On the other hand, where stagnant air arrives from the plenum chamber it is seen to spread laterally over the test section wall in a way suggesting that the jet from the slot - if indeed it is a jet - is deflected by the high momentum air farther out in the boundary layer.

Other oil flow pictures have been taken on the interior walls of the slots. As long as the flow goes from the test section into the plenum chamber it appears to be attached. If there is any flow separation at the entrance edge of the slot it must be restricted to a very short bubble. The pictures leave little doubt that the flow is for all practical purposes attached.

A preliminary survey of the total pressure of the flow at the centre of the slot has shown that the air entering it from the test section has nearly the full total pressure. However, when the wall boundary layer was thickened artificially (from 11 to 23 mm), then there was a noticeable reduction of total pressure, indicating that the present ratio of slot size to boundary layer thickness is just about small enough for the wall boundary layers to begin to make their presence felt. - Farther downstream, where air from the plenum chamber enters the slot, the total pressure was very much lower, as one would expect.

4.2 Measurements in the plenum chamber

Using the movable combined total-pressure and flow-direction probe, as well as conventional total pressure probes, the jet emerging from the test section into the plenum chamber has been investigated in some detail. The interpretation of the data in terms of velocities depends upon the value of the static pressure in the jet, and in view of the strong secondary flow in the plenum chamber it was not considered safe to take this pressure equal to the nominal plenum pressure from the tunnel calibration. Indeed, the pressure on the slat in the plenum chamber (Fig. 6) is considerably lower close to the jet than at the centre. Therefore the static pressure in the jet was taken to be the pressure, p_p, measured on the slat 1.5 mm from the slot edge.

As pointed out earlier the action of a slot is intimately connected with its longitudinal momentum flux. Part of this flux is located at the jet exit; according to theory (see Appendix) the 'effective' depth of this co-acting part of the jet is about 20 % of the slot width. For this reason it was thought proper to make the measurements of the transverse velocity component at the corresponding distance (0.8 mm) from the wall. Therefore, since the movable probe cannot be operated as close as that, the data obtained with this probe have been translated obliquely in the x-direction according to the measured flow inclination down to the attempted level.

The transverse velocity component v_p thus determined at the centre of the jet is shown in Fig. 7. Smooth curves have been drawn in to be used in the subsequent evaluation of the slot flow. It is noteworthy that the influence of the model (located between x = 0 and x = 90 mm) is felt rather far upstream and that it does not add very much to the outflow already present in the empty test section. The velocity level is clearly high enough in both cases for the quadratic term in the pressure equation to be significant. At the rear of the model v_p approaches zero, making the flow-direction measurement useless as a determination of the flow direction inside the slot. Oil flow pictures indicate that in the present case (which is somewhat different from the case shown in Fig. 5) there is no, or only
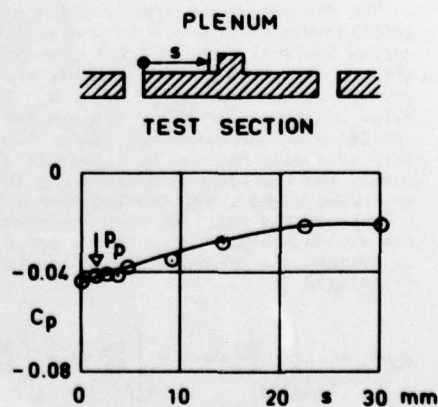

Fig. 6. Pressure distribution in plenum chamber on slat, in plane through the leading edge of the model.
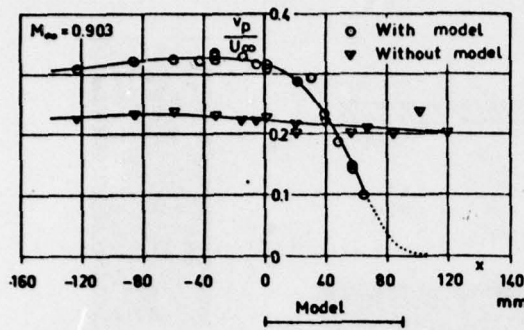
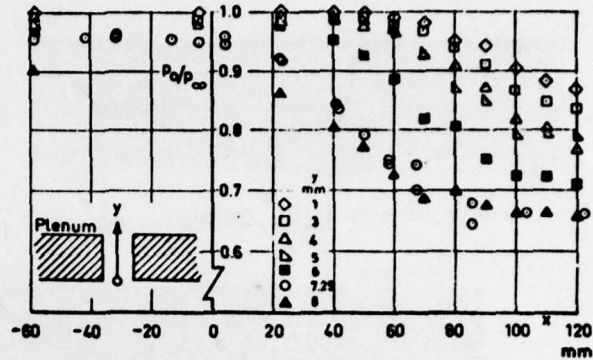Fig. 7. The transverse velocity component at the centre of the jet.

Fig. 8. The total pressure at the centre of the jet and slot (model present).

very slight, back-flow of air from the plenum chamber into the slot. The dotted part of the curve has been drawn accordingly, as a very crude guess.

The distribution of transverse velocity across the slot exit was found to be practically the same everywhere as long as the flow goes into the plenum chamber. No 'inviscid core' is visible, nor is there any indication of flow separation in the slot. With very little scatter the 'effective' width of the slot (transverse mass flux divided by the mass flux density at the centre) comes out as a' = 3.4 mm (or possibly 3.5 mm with the model present), i.e. 15 % smaller than the geometrical value a = 4.0 mm.

The total pressure variation along the jet with the model present is shown in Fig. 8. It is seen that the air going into the slot (at the centre) is effectively free-stream air and that it looses very little energy on its way through the slot. In the jet, on the other hand, the loss is considerable and this is also true, of course, towards the rear where the transverse velocity in the slot is reduced. When there is no model present the level of the total pressures (not shown) is somewhat reduced, while the losses towards the rear are absent. Altogether, this confirms that the slot flow under consideration is one with fairly small effects of inflow from the wall boundary layer and of viscous stress in the slot, and also that a reduction of the cross flow in the slot (as in three-dimensional tests) would soon make both these effects important.

4.3 A tentative flow model

In order to arrive at a quantitative description of the slot flow - and in particular to formulate a satisfactory wall boundary condition for the test section flow - we must establish a flow model which is simple enough to be amenable to mathematical analysis, yet complete enough to include all the essential features of our observations. A tentative empirical model to this end is indicated in Fig. 9. The flow pattern is as suggested by our observations. In particular, there is assumed to be no separation inside the slot.
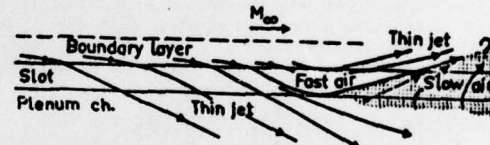
5. THE PRESSURE DIFFERENCE ACROSS THE WALL

The proposed flow model implies a specific relationship between the pressure difference across the wall and the transverse velocity through the slots, a relationship which can be checked against measurements, and, if verified, gives the basis for a homogeneous wall boundary condition of the classical type. The relationship is easily derived by integrating approximately the momentum equation along the path ① ② ③ specified in Fig. 10, leading from a point in the jet (or inside the slot) where the pressure first reaches the plenum pressure p_p, to the point on the wall where we measure the pressure p_w. Clearly (ds = element along ① ② ③)

ASSUMPTIONS:

(i) Inviscid flow in central part of slot.

(ii) Total pressure on each central streamline entering from test section known a priori.

(iii) Pressure in jet emerging from slot known a priori.

(iv) Effective slot width a' known a priori.

(v) Longitudinal momentum of air from the plenum chamber negligible.

Fig. 9. Tentative flow model.

$$p_w - p_p = \int_{①②③} \frac{\partial p}{\partial s} ds = -\int_{①} \frac{\partial p}{\partial y} dy + \int_{②} \frac{\partial p}{\partial z} dz + \int_{③} \frac{\partial p}{\partial y} dy,$$

where, neglecting the shear stress along the path,

$$\frac{\partial p}{\partial y} = -\varrho \left[(U+u)\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \right], \quad \frac{\partial p}{\partial s} = -\varrho \left[(U+u)\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \right].$$
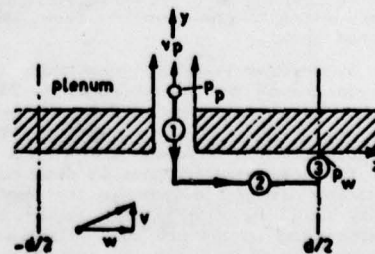

Fig. 10. Integration path in plane x = constant.

Here (u,v,w) is the perturbation velocity relative to an average central longitudinal velocity U in the slot region as specified in our model. Neglecting quadratic terms, except v^2 in the slot and the jet, and taking ρ to be a constant mean density in the slot region, we have approximately

$$① : \frac{\partial p}{\partial y} = -\rho\left(U\,\frac{\partial v}{\partial x} + \frac{\partial v^2/2}{\partial y}\right) , \quad ② : \frac{\partial p}{\partial z} = -\rho U\,\frac{\partial w}{\partial x} , \quad ③ : \frac{\partial p}{\partial y} = -\rho U\,\frac{\partial v}{\partial x} ,$$

giving

$$\frac{p_w - p_p}{\frac{1}{2}\rho_\infty U_\infty^2} = 2\,\frac{\rho}{\rho_\infty}\frac{U}{U_\infty}\,\frac{d}{dx}\left(C\,\frac{v_p}{U_\infty}\right) + \frac{\rho}{\rho_\infty}\left(\frac{v_p}{U_\infty}\right)^2 , \tag{4}$$

where

$$C = \int_① \frac{v}{v_p}\,dy - \int_② \frac{w}{v_p}\,dz - \int_③ \frac{v}{v_p}\,dy . \tag{5}$$

Thus our problem is reduced to estimating C. This is a reasonable task in view of the fact that we can expect v/v_p to be positive on ① and w/v_p to be negative on ② (while the contribution from the third integral must be small) so that fairly crude estimates for v/v_p and w/v_p might be sufficient. In fact, the main contribution is expected to come from ①, where v/v_p cannot be much different from unity. Therefore one should not be very much wrong in taking for C the value obtained from the slender body approximation. This value, from the analysis summarized in the Appendix, is with good approximation

$$C = a\left(\frac{1}{\pi}\,\ell n\,\frac{1}{\sin\frac{\pi}{2}\frac{a}{d}} + 0.46 + \ell/a\right) , \tag{6}$$

giving 11.1 mm with $a = 4$ mm and 10.6 mm with $a = a' = 3.4$ mm.

The pressure difference according to (4) has been computed using the smaller value for C and taking v_p and its derivative from the curve of Fig. 7. Values for ρ and U were obtained from the values for p_0 at the slot exit (see Fig. 8) at pressure p_p, assuming constant stagnation temperature. The result is shown in Fig. 11 (the shadow zone indicating the uncertainty) and compared with measured pressure differences. The good agreement might be fortuitous but is nevertheless very encouraging. It seems that one can conclude that the quadratic crossflow term in (4) is essential, that the assumption of attached flow in the slot is a good one, and that there is no effect present that calls for a viscous crossflow term of type included in Eq. (3).
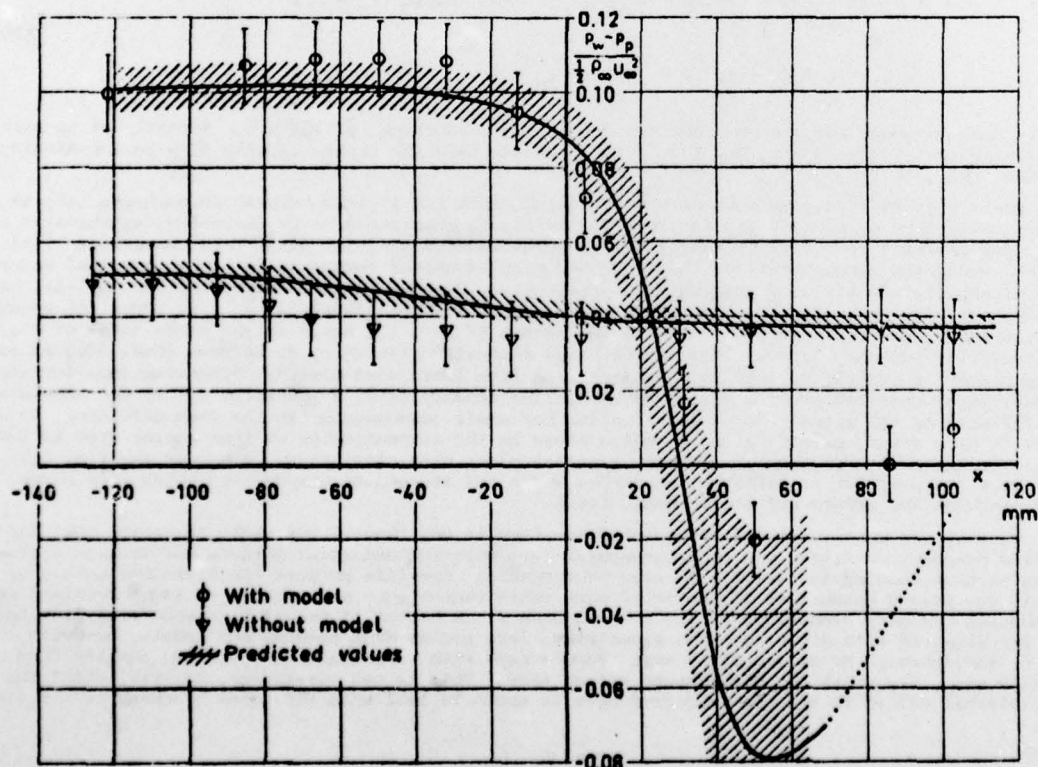

Fig. 11. Comparison of computed and measured pressure differences across the slotted wall.

The extension of the analysis to cases without a jet into the plenum chamber is straightforward. When the flow is reversed, it forms a jet into the test section and fills the slot with high momentum air to a depth $\ell_s < \ell$. Then $v = v_p$ at both ends of path ① , which now starts at $y = \ell_s$ and ends at $y = -0.22\,a$ (= the 'effective' depth of the co-acting part of the jet). Therefore, in this case, the quadratic crossflow term is cancelled while the estimate for C becomes $0.44 + \ell_s$. When the low momentum air penetrates into the test section the model implies that no pressure difference can be maintained and we have simply $p_w = p_p$. Finally, if the flow reverts to going outwards we have again Eq. (4), although as long as high momentum air fills the slot only to depth $\ell_s < \ell$ the value for C comes out slightly different, as shown in the Appendix. It remains of course to verify these predictions experimentally; this is presently under investigation.

6. THE MODIFIED HOMOGENEOUS BOUNDARY CONDITION

In order to collect the various pressure difference formulae into a homogeneous wall boundary condition we introduce the average crossflow velocity $\bar{v}$ (over $-d/2 < z < d/2$ in Fig. 10) normal to the wall and outside the wall boundary layer. Let the contribution to $\bar{v}$ from varying mass flux in the wall boundary layer be $\bar{v}_*$ ($= -U_\infty\,\partial\bar{\delta}_*/\partial x$ if the effect can be described in terms of a displacement thickness δ_*). Then the mass balance in the slender body approximation gives for the transverse slot velocity, as long as there is high momentum air in the slot,

$$\frac{v_p}{U_\infty} = \frac{U}{U_\infty}\frac{d}{a'}\,\frac{\bar{v} - \bar{v}_*}{U_\infty}\ ,\tag{7}$$

where U is the same average longitudinal velocity in the slot as before. We extend the function $v_p(x)$ to all values of x by prescribing $U = U_\infty$ and $\bar{v}_* = 0$ whenever they are not otherwise defined. Similarly we generalize the penetration depth ℓ_s as a function of x to be $\ell + 0.22\,a'$ whenever the slot is completely filled while elsewhere

$$\ell_s = \Big[\ell + 0.22\,a'\Big]_{x_0} + \int_{x_0}^{x}\frac{v_p}{U}\,dx,\quad x_0 \text{ being the closest upstream section completely filled.}\tag{8}$$

Furthermore let $\bar{p}$ be the corresponding average wall pressure. As shown in the Appendix $\bar{\delta}$ is obtained from the central wall pressure p_w by subtracting $0.22\,a'$ from C in Eq. (4). Hence, in terms of $K(\sim C \cdot d/a')$:

$$\frac{\bar{p} - p_p}{\frac{1}{2}\varrho_\infty U_\infty^2} = 2\,\frac{\varrho U}{\varrho_\infty U_\infty}\frac{d}{dx}\left(K\,\frac{U}{U_\infty}\,\frac{\bar{v}-\bar{v}_*}{U_\infty}\right) + \frac{\varrho}{\varrho_\infty}\left(\frac{U}{U_\infty}\frac{d}{a'}\frac{\bar{v}-\bar{v}_*}{U_\infty}\right)^2 \cdot \frac{\operatorname{sgn} v_p + 1}{2} \cdot \operatorname{sgn} K\ ,\tag{9}$$

with

$$\left.\begin{array}{l} K = d\left(\dfrac{1}{\pi}\ln\dfrac{1}{\sin\frac{\pi}{2}\frac{a}{d}} + 0.02 + \ell_s/a'\right)\ \text{if}\ >0\ \text{while}\ v_p \geq 0\ ;\\[3mm] K = d\left(0.22 + \ell_s/a'\right)\ \text{if}\ >0\ \text{while}\ v_p < 0\ ;\quad \text{else}\ K = 0. \end{array}\right\}\tag{10}$$

This is the proposed homogeneous boundary condition, connecting $\bar{p}$ with $\bar{v}$. Formally it is similar to the classical inviscid one, Eq. (1), with an added quadratic cross-velocity term in the manner of W.W. Wood (Ref. 16).

In reality it is of course much more complicated since (i) it is non-linear throughout, (ii) it is algebraically very complex, (iii) it contains additional quantities to be determined empirically or by further analysis and (iv) it can produce jump discontinuities in $\bar{p}(x)$ from continuous $\bar{v}(x)$. However, since the non-linearity of the transonic flow equations forces us to use a numerical method of solution, (i) and (ii) may not add much difficulty. Some of the quantities implied in (iii) have been demonstrated in Sections 4 and 5 to be manageable. As for computing $\bar{v}_*$, if important enough to be included, one could possibly do with the method of Ref. 17, based on the crude model of a constant velocity boundary layer. This approach was recently extended by P. Löfgren (Ref. 18), as part of the present research, to include classical slot flow (zero slot depth). Otherwise some turbulent boundary layer computer program has to be used. The anomaly (iv), essentially due to the non-uniform validity of the slender body approximation for small wavelengths in the flow direction, is not likely to give errors outside a small region close to the discontinuity as long as the flow is subsonic. In supersonic regions errors may propagate along characteristics, but since the flow equations have the property of diffusing expansion waves and steepening compression waves into shock waves, perhaps the errors may still remain local.

In view of its unconventional and complicated form it was thought advisable to verify that the modified boundary condition can be incorporated into existing numerical methods for transonic flow fields without causing instability or slow convergence. For this purpose the iterative method of Ref. 19 was chosen because of its normally very rapid convergence and because of its convenient way of handling boundary conditions at the outer boundary. A number of cases of axisymmetric flow have been investigated with different slot geometries, free stream Mach numbers and models, but with $\bar{v}_* = 0$ throughout. No difficulties were found except with very small slot widths, smaller than normally used, for which the convergence became slow. This is not surprising, however, since the computational method in the form employed here is known to fail when the outer boundary is a solid wall.

7. CONCLUDING REMARKS

The next step will be to extend the experimental verification of the flow model and its consequences for the wall pressure difference to a larger variety of slot flows: very likely this will lead to further modifications and perhaps to an accurate homogeneous boundary condition, the objective of our research. With this basis one can then employ accurate numerical methods to compute three-dimensional transonic flows about models in slotted test sections, be it to obtain wall interference corrections, or to adjust the slot geometry so as to eliminate the wall interference.

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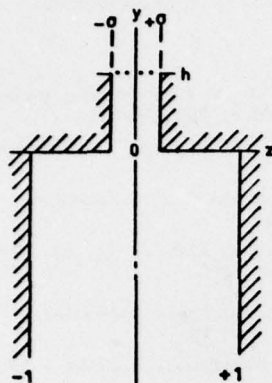
Appendix

APPROXIMATE ANALYSIS OF INVISCID SLOT FLOW

The classical slender body approximation for slot flow when applied to a rectangular slot with a jet into the plenum chamber, leads to the following problem for the harmonic function $P(z,y)$ in a doubly infinite strip (see sketch):

$$\frac{\partial^2 P}{\partial z^2} + \frac{\partial^2 P}{\partial y^2} = 0 \ ,$$

$$\Big\{ : \ \frac{\partial P}{\partial n} = 0 \ , \quad \Big| : \ P = 0 \ , \qquad \Bigg\} \qquad (A-1)$$

$$y \to -\infty \ : \ \frac{\partial P}{\partial y} \to 1 \ .$$

In the absence of a jet, with the constant pressure surface spanning the slot (.... in sketch), we have instead to find P in the semi-infinite strip $y < h$ with $P = 0$ on ..... We shall treat both problems simultaneously.

In terms of P the pressure at a point (y,z), when the velocity in the y-direction at $y \to -\infty$ is $\bar{v}(x)$, is obtained from

$$p-p_p = -\ Q_\infty\ U_\infty\ \frac{d}{2}\ \frac{d\bar{v}}{dx} \cdot P(z,y) + \frac{1}{2}\ Q_\infty\left(\frac{\bar{v}}{\sigma}\right)^2 \qquad (A-2)$$

with $\sigma = a/d$, $h = 2\ell/d$ or $2\ell_s/d$ (see Fig. 1 and Sec. 5). In particular the value of $P(1,0)$ is required when computing the pressure along the centre line of slats. The coefficient K in the homogeneous boundary condition (1) is obtained from

$$K = \frac{d}{2} \lim_{y \to -\infty} (y-P) \ . \qquad (A-3)$$

ζ-plane

The determination of P is accomplished by mapping conformally our strip in the plane $Z = z + iy$ on the lower half of the plane $\zeta = \xi + i\eta$ (see sketch) according to

$$dZ = -\ i\ \frac{2}{\pi}\ \sqrt{\frac{\zeta^2 - \sigma^2}{\zeta^2 - 1}}\ \frac{d\zeta}{\zeta} \ , \qquad (A-4)$$

placing the upper end of the strip at the origin and the lower end at infinity.

In the first case, with $P = 0$ at $---$, the result is

$$P(1,0) = -\ \frac{2}{\pi}\ \ell n\ \frac{1+ \sqrt{1-\xi_h^2}}{\xi_h} \ , \quad \lim_{y \to -\infty}(y-P) = \frac{1}{\pi}\left[\sigma\ \ell n\ \frac{1+\sigma}{1-\sigma} + \ell n\ (1-\sigma^2) - \ell n\ \xi_h^2 \right] , \qquad (A-5)$$

where ξ_h is obtained from solving the equation

$$\sigma\ \ell n\ \frac{\sigma t+1}{\sigma t-1} - \ell n\ \frac{t+1}{t-1} = \pi h \ , \quad t > 1/\sigma \ ; \quad \xi_h = \sqrt{\frac{\sigma^2 t^2 - 1}{t^2 - 1}} \ . \qquad (A-6)$$

In the second case, with $P = 0$ at ..... , we take ..... to be an ellipse with the major axis ξ_h and the minor axis η_h : the corresponding curve in the Z-plane cannot be much different from the line segment $y = h$. The result is

$$P(1,0) = -\ \frac{2}{\pi}\ \sigma\ \ell n\ \frac{1+\sqrt{1-\xi_h^2 + \eta_h^2}}{\xi_h + \eta_h} \ , \quad \lim_{y \to -\infty}(y-P) = \frac{1}{\pi}\left[\sigma\ \ell n\ \frac{1+\sigma}{1-\sigma} + \ell n(1-\sigma^2) - \ell n(\xi_h+\eta_h)^2 \right], \qquad (A-7)$$

where η_h is obtained from solving the equation

$$\sigma\ \ell n\ \frac{1+\sigma t}{1-\sigma t} - \ell n\ \frac{t+1}{t-1} = \pi h \ , \quad 1 < t < 1/\sigma \ ; \quad \eta_h = \sqrt{\frac{1 - \sigma^2 t^2}{t^2 - 1}} \ . \qquad (A-8)$$

A comprehensive set of computations has been performed using these formulae. The detailed results are not given here, however, since it turns out that they can be summarized in the following approximate formulae, valid with good accuracy for $\ell/a > 0.02$, $a/d < 0.2$:

$$K = d\left(\frac{1}{\pi}\ \ell n\ \frac{1}{\sin \frac{\pi}{2}\frac{a}{d}} + 0.02 + \begin{Bmatrix}\ell/a + 0.22 \\ \ell_s/a\end{Bmatrix}\right) \cdot \begin{Bmatrix}\text{with jet} \\ \text{no jet}\end{Bmatrix} ; \ P(1,0) = -2\left(\frac{K}{d} + 0.22\right). \qquad (A-9)$$

Here ℓ_s is the depth of penetration of the flow into the slot when there is no jet. The formula for K is an improvement upon that of Eq. (2).

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
A theoretical and experimental study is in progress of the flow through slotted
walls under a variety of conditions. The ultimate objective is to make possible
accurate numerical computation of transonic flows around models in slotted test
sections. This paper is concerned with a slot flow configuration typical of two-
dimensional, low lift tests at high subsonic free stream Mach numbers. With the
test section empty the slot flow is outwards, into the plenum chamber, and this
remains true over a large part of the test section when the model is introduced.
From oilflow pictures and pressure measurements in and around the slots it is ———

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concluded for the configurations investigated that the slot flow is slightly influenced by the presence of the wall boundary layer, that the flow within the slot is attached and approximately inviscid although influenced by boundary layer formation, that the flow enters the plenum chamber as a thin free jet and that the transverse velocity in the jet and slot is too large for a linear pressure drop equation to be sufficiently accurate. When the slot flow turns back over the rear end of the model it may admit stagnant air from the plenum chamber into the test section; the ability of the slot to maintain a pressure difference across the wall is then necessarily reduced. Based on these observations a tentative flow model is proposed, yielding a relationship between the pressure difference across the wall and the transverse velocity through the slots. It agrees well with the measurements a far as they go. The corresponding homogeneous wall boundary condition is of the classical inviscid type with an added quadratic cross-velocity term in the manner of W W Wood. Improved expressions for the coefficient of the streamline curvature term are obtained, accounting for the depth of the slot and the presence of a jet. The new boundary condition has been demonstrated to be compatible with computing numerically the transonic flow field around a model in the test section.